

Exercise 43

Evaluate the integral.

$$\int_0^{\pi} f(x) dx \quad \text{where } f(x) = \begin{cases} \sin x & \text{if } 0 \leq x < \pi/2 \\ \cos x & \text{if } \pi/2 \leq x \leq \pi \end{cases}$$

Solution

Split up the integral over the intervals on which the given function is defined. Then use the second part of the fundamental theorem of calculus to evaluate the integrals.

$$\begin{aligned} \int_0^{\pi} f(x) dx &= \int_0^{\pi/2} f(x) dx + \int_{\pi/2}^{\pi} f(x) dx \\ &= \int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \cos x dx \\ &= \int_0^{\pi/2} -\frac{d}{dx}(\cos x) dx + \int_{\pi/2}^{\pi} \frac{d}{dx}(\sin x) dx \\ &= -\int_0^{\pi/2} \frac{d}{dx}(\cos x) dx + \int_{\pi/2}^{\pi} \frac{d}{dx}(\sin x) dx \\ &= -(\cos x) \Big|_0^{\pi/2} + (\sin x) \Big|_{\pi/2}^{\pi} \\ &= -\left(\cos \frac{\pi}{2} - \cos 0\right) + \left(\sin \pi - \sin \frac{\pi}{2}\right) \\ &= -(0 - 1) + (0 - 1) \\ &= 0 \end{aligned}$$